# Chapter 8.

## 8.1 Graphing Linear Functions.

Slope-intercept form: $y = mx + b$

$m$ is the slope, the $y$-intercept is $(0, b)$

Point-slope form: $y - y_1 = m(x - x_1)$

$m$ is the slope, the point is $(x_1, y_1)$

Horizontal line: $y = c$

slope is 0, the $y$-intercept is $(0, c)$

Vertical line: $x = c$

Equations of parallel lines have the same slope $m_1 = m_2$

Perpendicular lines $m_1 = -\frac{1}{m_2}$

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Exercise: P516 #7, 24, 35, 39, 41, 46. P518 80, 83

## 8.2 Reviewing Function Notation and Graphing Nonlinear Functions.

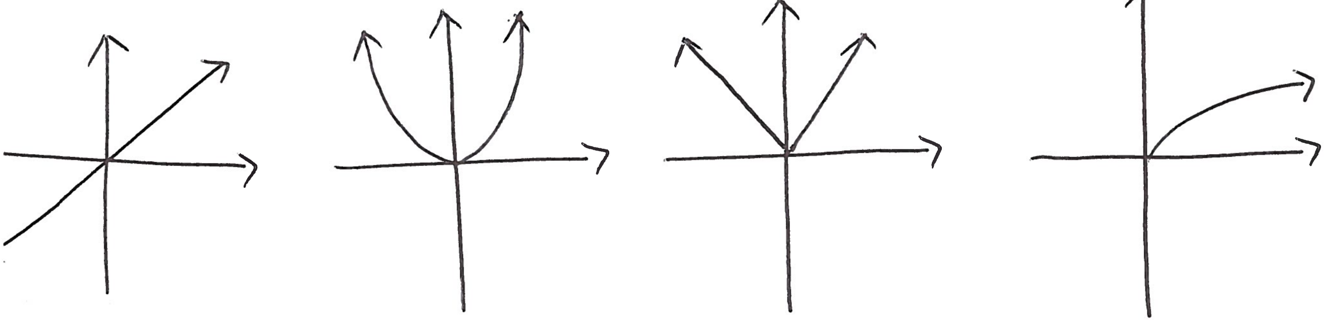
Each function graphed is the graph of a function and passes the vertical line test.

To graph the square root functions, we need to identify the domain, and then evaluate the function for several values of $x$, plot the resulting points and connect the points with a smooth curve.

Exercise: P525 29, 32, 41, 42. P526 44, 48

Common Graph:

$f(x) = x$ $\quad$ $f(x) = x^2$ $\quad$ $f(x) = |x|$ $\quad$ $f(x) = \sqrt{x}$



Vertical shift: (Let K be a positive number)

| Graph of | Moved |
|---|---|
| $g(x) = f(x) + K$ | K units upward |
| $g(x) = f(x) - K$ | K units downward |

Horizontal shift: (Let h be a positive number)

| Graph of | Moved |
|---|---|
| $g(x) = f(x-h)$ | h units to the right |
| $g(x) = f(x+h)$ | h units to the left |

Reflection about the x-axis.

The graph of $g(x) = -f(x)$ is the graph of $f(x)$ reflected about x-axis.

8.4 Variation and Problem Solving

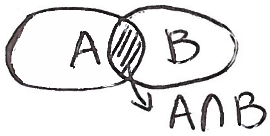
- y varies directly as x, or y is directly proportional to x: $y = kx$
- y varies inversely as x: $y = \frac{k}{x}$
- y varies jointly as the other variables: $y = kxz$. (* k is constant)

Exercise: P542 #8, 17, 25 P543 #39, 44.

# Chapter 4

## 9.1 Compound Inequalities

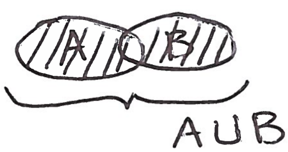
Intersection of two sets, A and B is the set of all elements common to both sets. A intersect B is denoted by $A \cap B$.




| Compound Inequality | Compact Form | Interval Notation |
|---|---|---|
| $3 \le X$ and $X \le 8$ | $3 \le X \le 8$ | $[3, 8]$ |

Graph:


Union of two sets, A or B, is the set of elements that belong to either of the sets. A union B is denoted by $A \cup B$




$x \le 1$ or $X \ge 3$

$X \le 1$  $(-\infty, 1]$

$X \ge 3$  $[3, \infty)$



$X \le 1 \cup X \ge 3$  $(-\infty, 1] \cup [3, \infty)$



## 9.2 Absolute Value Equations

Solving Equations of the form $|X| = a \iff X = a$ or $X = -a$

a is a positive number.

However, if a is a negative number, then $|X| = a$ has no solution.

Exercise P563 #5, 11, 53, 68.

After solving the absolute Value equations, we have to check our results by putting into original equations, because sometimes we might get a false statement.

9.3 Absolute Value Inequalities

$|x| < a$, a is positive number then $-a < x < a$. 


$|x| > a \iff x < -a$ or $x > a$. 


$|x| = a$ 

$x = a$, $x = -a$

Exercise P569 37, 59, 64, 73, 82.

9.4 Graphing Linear Inequalities in Two Variables and Systems of Linear Inequalities.

$>$, $<$ graph a dashed boundary line (indicating that the points on the line are not solutions of the inequality)

$\geq$, $\leq$ graph a solid boundary line (indicating that the points on the line are solutions of the inequality)

When graphing an inequality, make sure the test point is substituted into the original inequality, if the test point is a true statement, then shade the half-plane that contains the test point, if it is a false statement, then shade the half-plane that does not contain the test point.

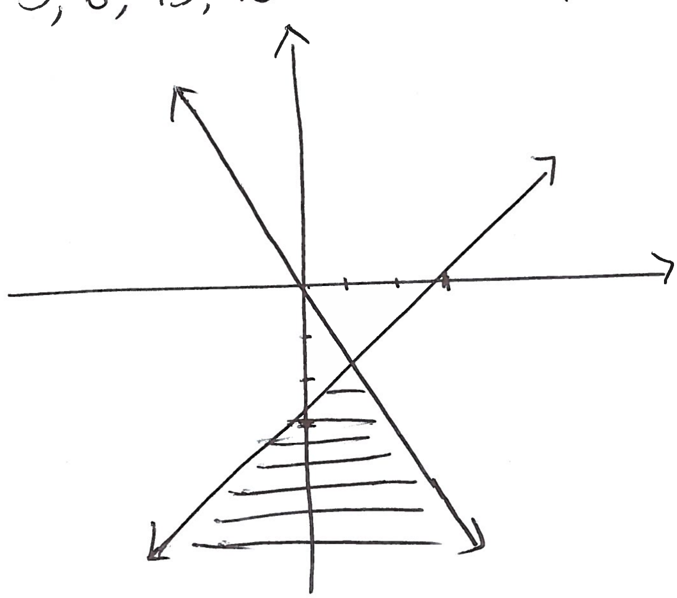
Graphing the Solution Region of a System of Linear Inequalities

Step 1: Graph each inequality in the system on the same set of axes.

Step 2: The solutions of the system are the points common to the graphs of all the inequalities in the system.

Exercise: P577 3, 6, 15, 16 P578 32, 59, 62, 77, 80, 85.

$$\begin{cases} x - y \geq 3 \\ y \leq -2x \end{cases}$$



Chapter 10

$\sqrt{a}$ → radicand, $\sqrt{\ }$ → radical sign

10.1 Radicals and Radical Functions

Principal and Negative Square Roots

If $a$ is a positive number, then

$\sqrt{a}$ is the principal, or nonnegative, square root of $a$. $(\sqrt{36}=6)$

$-\sqrt{a}$ is the negative square root of $a$. $(-\sqrt{36}=-6)$

· $\sqrt{0}=0$, $\sqrt{1}=1$

· $\sqrt{-9}$ is not a real number, $\sqrt[3]{-64}=-4$, $\sqrt[3]{-27x^9}=-3x^3$

$\sqrt[4]{-15}$ is not a real number, $\sqrt[5]{-32}=-2$, $\sqrt[3]{1}=1$

Finding $\sqrt[n]{a^n}$ $\begin{cases} \text{If } n \text{ is an even positive integer, then } \sqrt[n]{a^n}=|a| \\ \text{If } n \text{ is an odd positive integer, then } \sqrt[n]{a^n}=a \end{cases}$

Finding the domain of $\begin{cases} f(x)=\sqrt{x-5} & x\geq 5 \quad [5,\infty) \\ g(x)=\sqrt[3]{x+8} & (-\infty,\infty)\ \mathbb{R}. \end{cases}$

Exercise: P593 # 25, 28 P594–595 # 38, 53, 68, 90, 91, 101, 102

10.2 Rational Exponents

If $n$ is a positive integer greater than 1 and $\sqrt[n]{a}$ is a real number, then

$a^{\frac{1}{n}}=\sqrt[n]{a}$

If $m$ and $n$ are positive integers greater than 1 with $\frac{m}{n}$ in simplest form then $a^{\frac{m}{n}}=\sqrt[n]{a^m}=(\sqrt[n]{a})^m$ as long as $\sqrt[n]{a}$ is a real number.

$a^{-\frac{m}{n}}=\frac{1}{a^{\frac{m}{n}}}$ as long as $a^{\frac{m}{n}}$ is a nonzero real number.

① $a^m\cdot a^n=a^{m+n}$ ② $(a^m)^n=a^{m\cdot n}$ ③ $(ab)^n=a^nb^n$ ④ $\left(\frac{a}{c}\right)^n=\frac{a^n}{c^n}$, $c\neq 0$

⑤ $\frac{a^m}{a^n}=a^{m-n}$, $a\neq 0$ ⑥ $a^0=1$, $a\neq 0$ ⑦ $a^{-n}=\frac{1}{a^n}$, $a\neq 0$

Exercise: P601–602 # 11, 16, 19, 28, 38, 40, 51, 54, 59, 81, 89, 92.

0.3 Simplifying Radical Expressions

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then $\sqrt[n]{a}\cdot\sqrt[n]{b}=\sqrt[n]{ab}$

If $\sqrt[n]{b}$ is not zero, then $\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Don't forget $5\sqrt{2}$ means $5\cdot\sqrt{2}$

Distance Formula: $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ if we are given two points $(x_1,y_1)$ and $(x_2,y_2)$.

Midpoint Formula: $\left(\frac{x_1+x_2}{2},\ \frac{y_1+y_2}{2}\right)$

Exercise: P609-611 # 4, 10, 12, 18, 30, 49, 54, 60, 70, 74, 84, 95

0.4 Adding, Subtracting and Multiplying Radical Expressions

Like Radicals: Radicals with the same index and the same radicand are like radicals.

Exercise P615-616 # 7, 10, 13, 16, 21, 25, 26, 30, 34, 50, 67.

0.5 Rationalizing Denominators and Numerators of Radical Expressions.

example: $\sqrt{\frac{7x}{3y}}=\frac{\sqrt{7x}}{\sqrt{3y}}\cdot\frac{\sqrt{3y}}{\sqrt{3y}}=\frac{\sqrt{21xy}}{3y}$

$$\frac{\sqrt[4]{x}}{\sqrt[4]{81y^5}}=\frac{\sqrt[4]{x}}{\sqrt[4]{81y^4}\cdot\sqrt[4]{y}}=\frac{\sqrt[4]{x}}{3y\sqrt[4]{y}}\cdot\frac{\sqrt[4]{y^3}}{\sqrt[4]{y^3}}=\frac{\sqrt[4]{xy^3}}{3y\sqrt[4]{y^4}}=\frac{\sqrt[4]{xy^3}}{3y^2}$$

$(a+b)(a-b)=a^2-b^2$

Ex:
$$\frac{5}{\sqrt{3}-2}=\frac{5}{\sqrt{3}-2}\cdot\frac{\sqrt{3}+2}{\sqrt{3}+2}=\frac{5(\sqrt{3}+2)}{(\sqrt{3})^2-2^2}=\frac{5\sqrt{3}+10}{3-4}=-5\sqrt{3}-10$$

Rationalize the numerator of $\frac{\sqrt[3]{2x^2}}{\sqrt[3]{5y}}$

$$\frac{\sqrt[3]{2x^2}}{\sqrt[3]{5y}}=\frac{\sqrt[3]{2x^2}\cdot\sqrt[3]{4x}}{\sqrt[3]{5y}\cdot\sqrt[3]{4x}}=\frac{\sqrt[3]{8x^3}}{\sqrt[3]{20xy}}=\frac{2x}{\sqrt[3]{20xy}}$$

$$\frac{\sqrt{x+2}}{5}=\frac{\sqrt{x+2}\cdot\sqrt{x+2}}{5\cdot\sqrt{x+2}}=\frac{x+2}{5\sqrt{x+2}}$$

Exercise: P622-623 # 11, 12, 18, 28, 32, 33, 39, 53, 62, 80, 86.

0.6 Radical Equations and Problem Solving

Steps: 1. Isolate one radical on one side of the equation

2. Raise each side of the equation to a power equal to the index of the radical and simply

3. If the equation still contains a radical term, repeat step 1 and 2. If not, solve the equation.

4. check all proposed solutions in the original equation.

Example: $\sqrt[3]{x+1}+5=3$

$\sqrt[3]{x+1}=-2$

$(\sqrt[3]{x+1})^3=(-2)^3$

$x+1=-8$

$x=-9$

check $x=-9$, $\sqrt[3]{-9+1}+5$

$=\sqrt[3]{-8}+5$

$=-2+5=3$ ✓

$\sqrt{4-x}=x-2$

$(\sqrt{4-x})^2=(x-2)^2$

$4-x=x^2-4x+4$

$x^2-3x=0$

$x(x-3)=0$

$x=0 \quad x=3$

check:

$x=0$ $\sqrt{4-0}=2\stackrel{?}{=}0-2$

$2\neq-2$ X

$x=3$ $\sqrt{4-3}\stackrel{?}{=}3-2$

$1=1$ ✓

$\sqrt{2x+5}+\sqrt{2x}=3$

$\sqrt{2x+5}=3-\sqrt{2x}$

$(\sqrt{2x+5})^2=(3-\sqrt{2x})^2$

$\cancel{2x}+5=9-6\sqrt{2x}+\cancel{2x}$

$6\sqrt{2x}=4$

$\sqrt{2x}=\frac{4}{6}=\frac{2}{3}$

$(\sqrt{2x})^2=(\frac{2}{3})^2$

$2x=\frac{4}{9}$

$x=(\frac{4}{9})(\frac{1}{2})=\frac{2}{9}$

check $x=\frac{2}{9}$

$\sqrt{2(\frac{2}{9})+5}+\sqrt{2(\frac{2}{9})}\stackrel{?}{=}3$

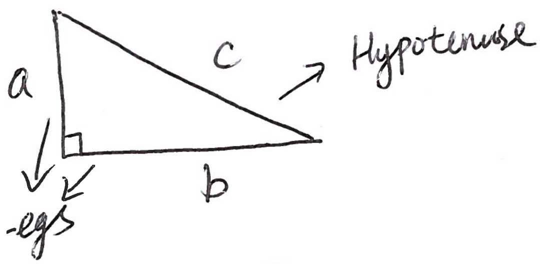
$\sqrt{\frac{4}{9}+5}+\sqrt{\frac{4}{9}}$

$=\sqrt{\frac{49}{9}}+\frac{2}{3}$

$=\frac{7}{3}+\frac{2}{3}=\frac{9}{3}=3$ ✓

Pythagorean Theorem

$a^2+b^2=c^2$




Exercise: P630−633 # 8, 21, 42, 54, 56, 60, 62, 75, 91, 96.

0.7. Complex Numbers

$i^2=-1$, $i=\sqrt{-1}$

xample: $\sqrt{-36}\cdot\sqrt{-1}=6i\cdot i=6i^2=-6$.

$\frac{\sqrt{-125}}{\sqrt{5}}=i\sqrt{25}=5i$

Complex Numbers is a number that can be written in the form $a+bi$, where a and b are real numbers.

If $a+bi$ and $c+di$ are complex numbers, then their sum is

$(a+bi)+(c+di)=(a+c)+(b+d)i$

their difference is

$(a+bi)-(c+di)=(a-c)+(b-d)i$

Example: $(2+3i)+(-3+2i)=(2-3)+(3+2)i=-1+5i$

$(2-5i)(4+i)=(2)(4)+2i-(5i)(4)-(5i)(i)=8-18i+5=13-18i$

$=8+2i-20i-5i^2$

$(a+bi)$ and $(a-bi)$ are called complex conjugates of each other, and

$(a+bi)(a-bi)=a^2-abi+abi-b^2i^2=a^2-b^2(-1)=a^2+b^2$

$i^1=i$ $\quad i^5=i$ $\quad i^9=i$

$i^2=-1$ $\quad i^6=-1$ $\quad i^{10}=-1$

$i^3=-i$ $\quad i^7=-i$ $\quad i^{11}=-i$

$i^4=1$ $\quad i^8=1$ $\quad i^{12}=1$

Example: $i^7=i^4\cdot i^3=(1)(-i)=-i$

$i^{20}=(i^4)^5=1^5=1$

$i^{46}=i^{44}\cdot i^2=(i^4)^{11}\cdot i^2=-1$

$i^{-12}=\frac{1}{i^{12}}=\frac{1}{(i^4)^3}=\frac{1}{1^3}=1$

Exercise P640−641 # 8, 16, 25, 30, 38, 46, 52, 64, 86, 88, 92, 111.

# Chapter 11

## 11.1 Solving Quadratic Equations by Completing the Square

$$2x^2 - 14 = 0$$
$$2x^2 = 14$$
$$x^2 = 7$$
$$x = \pm\sqrt{7}$$

$$(x+1)^2 = 12$$
$$x+1 = \pm\sqrt{12}$$
$$x+1 = \pm 2\sqrt{3}$$
$$x = -1 \pm 2\sqrt{3}$$

$$(2x-5)^2 = -16$$
$$2x-5 = \pm\sqrt{-16}$$
$$2x = 5 \pm 4i$$
$$x = \frac{5 \pm 4i}{2}$$

* $(a+b)^2 = a^2 + 2ab + b^2$, $(a-b)^2 = a^2 - 2ab + b^2$

$$(x+4)^2 = x^2 + 8x + 16$$
$$(x-3)^2 = x^2 - 6x + 9$$

$$(x-5)^2 = x^2 - 10x + 25$$
$$\left(\tfrac{1}{2}\right)(10) = 5 \qquad 5^2 = 25$$

EX: $4x^2 - 16x + 36 = 0$
$$x^2 - 4x + 9 = 0$$
$$x^2 - 4x = -9$$
$$x^2 - 4x + 2^2 = -9 + 2^2$$
$$\left(\tfrac{1}{2}\right)(4) = 2$$
$$x^2 - 4x + 4 = -5$$
$$(x-2)^2 = -5$$

$$\sqrt{(x-2)^2} = \pm\sqrt{-5}$$
$$x-2 = \pm 5i$$
$$x = \pm 5i + 2$$

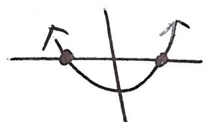
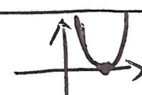
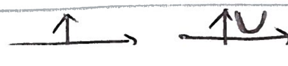
The solutions are $5i+2$ and $-5i+2$.

## 11.2. Solving Quadratic Equations by the Quadratic Formula.

$ax^2 + bx + c = 0$, $a \neq 0$ $\qquad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EX: $3x^2 + 16x + 5 = 0$

$$x = \frac{-16 \pm \sqrt{16^2 - 4(3)(5)}}{2(3)} = -\frac{1}{3}, -5$$

| $b^2 - 4ac$ | Number and Type of Solutions |
|---|---|
| Positive | Two real solutions  |
| zero | One real solution   |
| negative | No solution  |

11.3 Solving Equations by Using Quadratic Methods.

X: $\frac{3x}{x-2} - \frac{x+1}{x} = \frac{6}{x(x-2)}$ multiply the LCD on both sides

LCD = $x(x-2)$

$3x^2 - (x-2)(x+1) = 6$

$3x^2 - (x^2 - x - 2) = 6$

$3x^2 - x^2 + x + 2 = 6$

$2x^2 + x - 4 = 0$ → $x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-4)}}{2 \cdot 2} = \frac{-1 \pm \sqrt{33}}{4}$

X: $p^4 - 3p^2 - 4 = 0$

$(p^2 - 4)(p^2 + 1) = 0$

$(p-2)(p+2)(p^2+1) = 0$

$p = 2 \quad p = -2 \quad p = \pm i$

Also we can use substitution method,

Let $x = p^2$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4 \quad x = -1$

$p^2 = 4 \quad p = \pm 2$

$p^2 = -1 \quad p = \pm i$

* Distance = (Time)(Speed) $d = rt$